Non-boring cryptography

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Learning Objectives

- Blind signatures and applications
- Homomorphic encryption
- Secure multiparty computation (SMC) theory
- Common SMC adversary models
- Specialized protocols for SMC problems

Reminder: RSA

Pick p, q prime and e such that

$$GCD((p-1)(q-1), e) = 1 \tag{1}$$

- Define $n = pq$,
- compute d such that $ed \equiv 1 \mod (p-1)(q-1)$.
- Let $s := m^d \mod n$.
- Then $m \equiv s^e \mod n$.

RSA Summary

- Public key: n, e
- Private key: $d \equiv e^{-1} \mod \phi(n)$ where $\phi(n) = (p-1) \cdot (q-1)$
- Encryption: $c \equiv m^e \mod n$
- Decryption: $m \equiv c^d \mod n$
- Signing: $s \equiv m^d \mod n$
- Verifying: $m \equiv s^e \mod n$?

Low Encryption Exponent Attack

- e is known
- M maybe small
- $C = M^e < n$?
- If so, can compute $M = \sqrt[e]{C}$

$\Rightarrow$ Small e can be bad!

Padding and RSA Symmetry

- Padding can be used to avoid low exponent issues (and issues with $m = 0$ or $m = 1$)
- Randomized padding defeats chosen plaintext attacks
- Padding breaks RSA symmetry:

$$D_{A_{priv}}(D_{B_{priv}}(E_{A_{pub}}(E_{B_{pub}}(M)))) \neq M \tag{2}$$

- PKCS#1 / RFC 3447 define a padding standard

Blind signatures with RSA

1. Obtain public key
   (e, n)
2. Compute
   $f := FDH(m)$,
   $f < n$.
3. Pick blinding factor
   $b \in \mathbb{Z}_n$
4. Transmit
   $f' := fb^e \mod n$

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Break

A Social Problem

This was a question posed to RAND researchers in 1971:

> *"Suppose you were an advisor to the head of the KGB, the Soviet Secret Police. Suppose you are given the assignment of designing a system for the surveillance of all citizens and visitors within the boundaries of the USSR. The system is not to be too obtrusive or obvious. What would be your decision?"*

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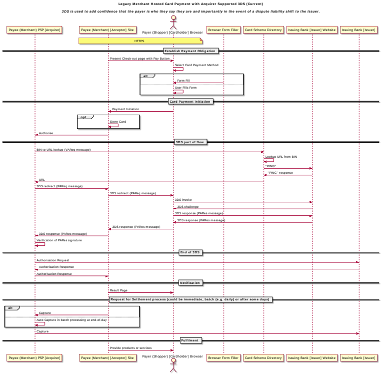
Mastercard/Visa are too transparent.

"I think one of the big things that we need to do, is we need to get a way from true-name payments on the Internet. The credit card payment system is one of the worst things that happened for the user, in terms of being able to divorce their access from their identity." –Edward Snowden, IETF 93 (2015)

The Bank's Problem

3D secure ("verified by visa") is a nightmare:

- Complicated process
- Shifts liability to consumer
- Significant latency
- Can refuse valid requests
- Legal vendors excluded
- No privacy for buyers



Online credit card payments will be replaced, but with what?

The Bank's Problem

- Global tech companies push oligopolies
- Privacy and federated finance are at risk
- Economic sovereingity is in danger

Do you want to live under total surveillance?

Digital cash, made **socially responsible**.

⟨Taler⟩

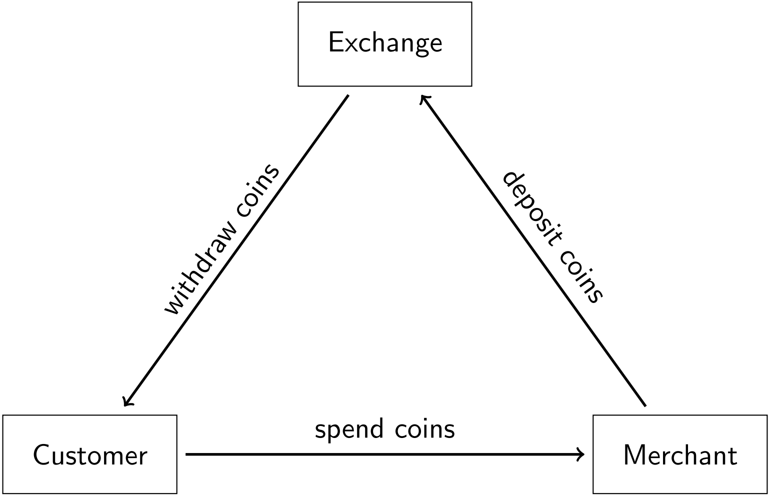
Privacy-Preserving, Practical, Taxable, Free Software, Efficient

What is Taler?

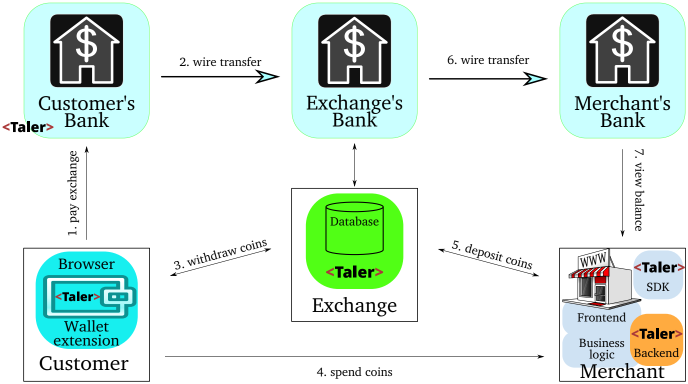
Taler is an electronic instant payment system.

- Uses electronic coins stored in **wallets** on customer's device
- Like **cash**
- Pay in **existing currencies** (i.e. EUR, USD, BTC), or use it to create new **regional currencies**

Taler Overview



Architecture of Taler



$\Rightarrow$ Convenient, taxable, privacy-enhancing, & resource friendly!

Usability of Taler

`https://demo.taler.net/`

1. Install Chrome extension.
2. Visit the `bank.demo.taler.net` to withdraw coins.
3. Visit the `shop.demo.taler.net` to spend coins.

Taxability

We say Taler is taxable because:

- Merchant's income is visible from deposits.
- Hash of contract is part of deposit data.
- State can trace income and enforce taxation.

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Limitations:

- withdraw loophole
- *sharing* coins among family and friends

How does it work?

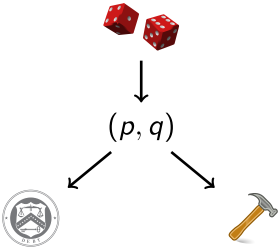
We use a few ancient constructions:

- Cryptographic hash function (1989)
- Blind signature (1983)
- Schnorr signature (1989)
- Diffie-Hellman key exchange (1976)
- Cut-and-choose zero-knowledge proof (1985)

But of course we use modern instantiations.

Exchange setup: Create a denomination key (RSA)

1. Pick random primes p, q.
2. Compute $n := pq$,
   $\phi(n) = (p-1)(q-1)$
3. Pick small $e < \phi(n)$ such that $d := e^{-1} \mod \phi(n)$ exists.
4. Publish public key (e, n).



Merchant: Create a signing key (EdDSA)

- pick random $m \mod o$ as private key
- $M = mG$ public key

$$\downarrow$$

$$m$$

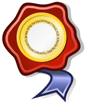
$$\downarrow$$

$$\mathrm{M}$$

Capability: $m \Rightarrow$ 

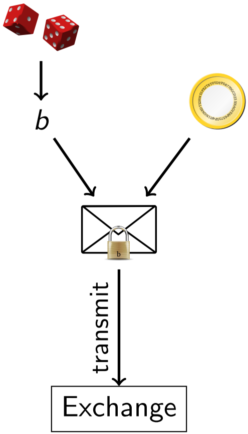
Customer: Create a planchet (EdDSA)

- Pick random $c \mod o$ private key
- $C = cG$ public key

Capability: $c \Rightarrow$ 

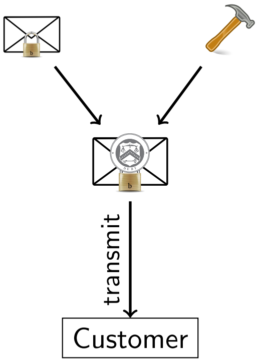
Customer: Blind planchet (RSA)

1. Obtain public key (e, n)
2. Compute $f := FDH(C)$, $f < n$.
3. Pick blinding factor $b \in \mathbb{Z}_n$
4. Transmit $f' := fb^e \mod n$



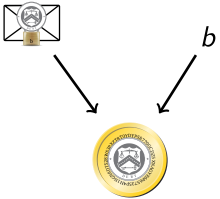
Exchange: Blind sign (RSA)

1. Receive f'.
2. Compute $s' := f'^d \mod n$.
3. Send signature s'.

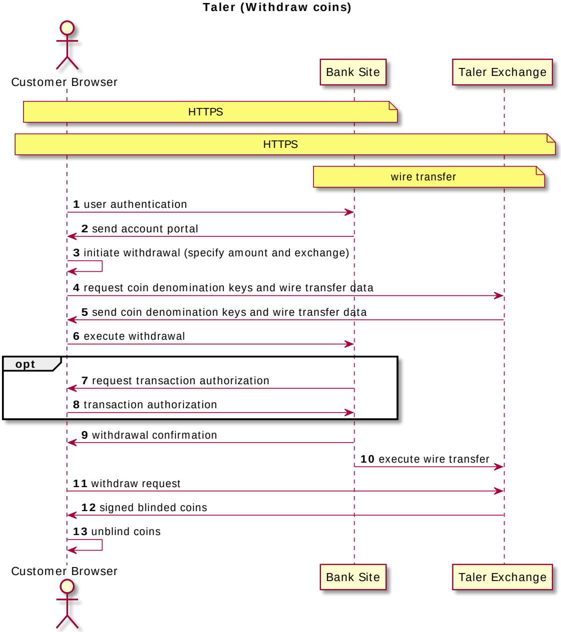


Customer: Unblind coin (RSA)

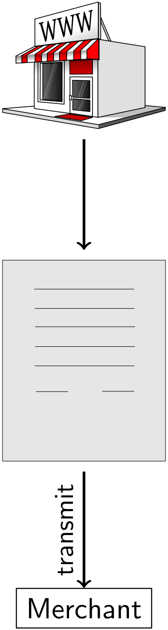
1. Receive s'.
2. Compute $s := s' b^{-1} \mod n$



Withdrawing coins on the Web

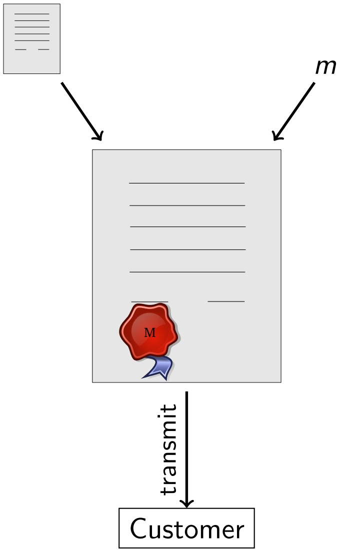


Customer: Build shopping cart



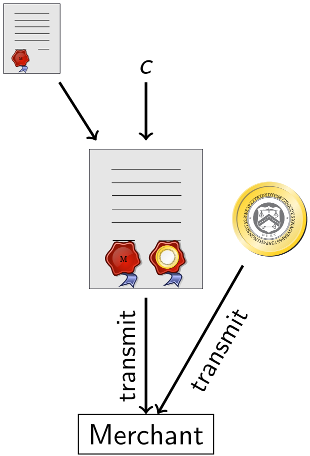
Merchant: Propose contract (EdDSA)

1. Complete proposal D.
2. Send D, $EdDSA_m(D)$



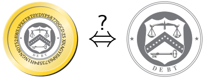
Customer: Spend coin (EdDSA)

1. Receive proposal D, $EdDSA_m(D)$.
2. Send s, C, $EdDSA_c(D)$

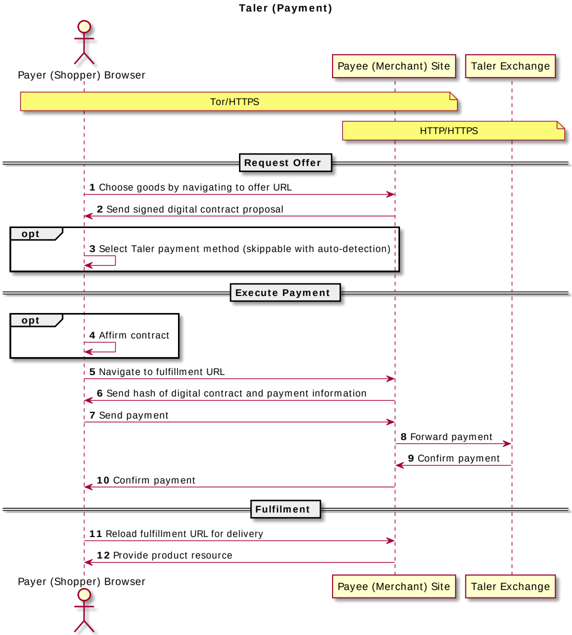


Merchant and Exchange: Verify coin (RSA)

$$s^e \stackrel{?}{\equiv} m \mod n$$



Payment processing with Taler



Break

Giving change

It would be inefficient to pay EUR 100 with 1 cent coins!

- Denomination key represents value of a coin.
- Exchange may offer various denominations for coins.
- Wallet may not have exact change!
- Usability requires ability to pay given sufficient total funds.

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Key goals:

- maintain unlinkability
- maintain taxability of transactions

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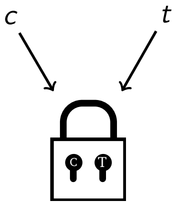
Method:

- Contract can specify to only pay *partial value* of a coin.
- Exchange allows wallet to obtain *unlinkable change* for remaining coin value.

Diffie-Hellman (ECDH)

1. Create private keys $c, t \bmod o$
2. Define $C = cG$
3. Define $T = tG$
4. Compute DH $cT = c(tG) = t(cG) = tC$

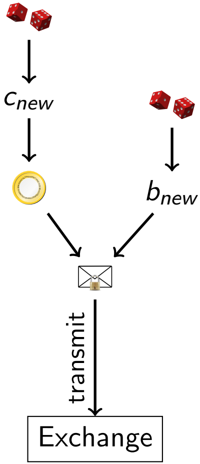


Strawman solution

Given partially spent private coin key c_{old}:

1. Pick random $c_{new} \mod o$ private key
2. $C_{new} = c_{new} G$ public key
3. Pick random b_{new}
4. Compute $f_{new} := FDH(C_{new})$, $m < n$.
5. Transmit $f'_{new} := f_{new} b^e_{new} \mod n$

... and sign request for change with c_{old}.



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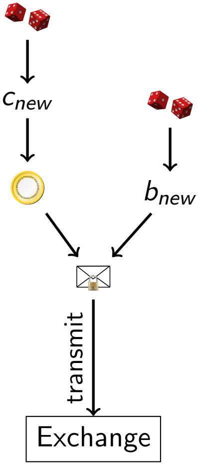
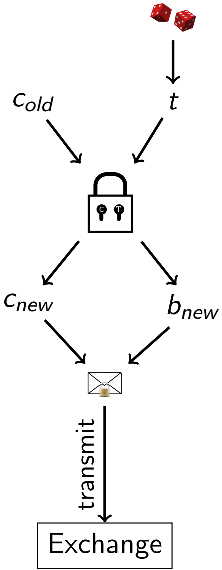


Problem: Owner of c_{new} may differ from owner of c_{old}!

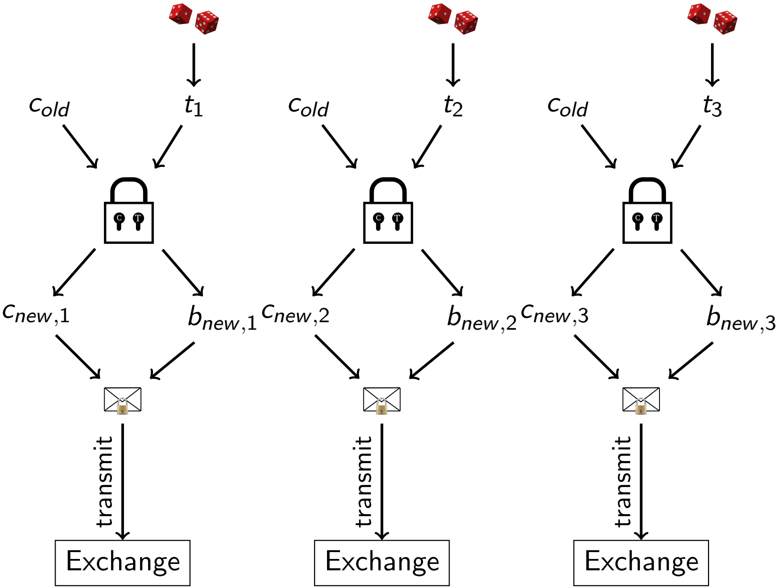
Customer: Transfer key setup (ECDH)

Given partially spent private coin key c_{old}:

1. Let $C_{old} := c_{old} G$ (as before)
2. Create random private transfer key t mod o
3. Compute $T := tG$
4. Compute $X := c_{old}(tG) = t(c_{old} G) = tC_{old}$
5. Derive c_{new} and b_{new} from X
6. Compute $C_{new} := c_{new} G$
7. Compute $f_{new} := FDH(C_{new})$
8. Transmit $f'_{new} := f_{new} b_{new}^e$



Cut-and-Choose



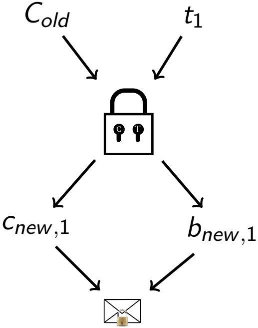
Exchange: Choose!

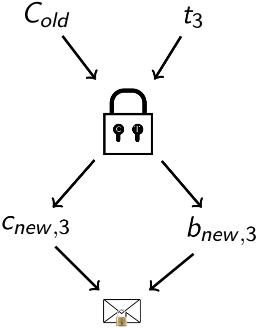
Exchange sends back random $\gamma \in \{1, 2, 3\}$ to the customer.

Customer: Reveal

1. If $\gamma = 1$, send t_2, t_3 to exchange
2. If $\gamma = 2$, send t_1, t_3 to exchange
3. If $\gamma = 3$, send t_1, t_2 to exchange

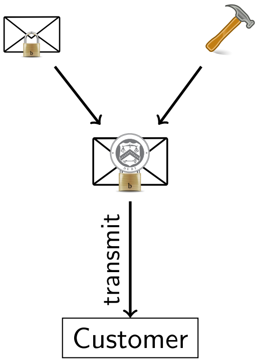
Exchange: Verify ($\gamma = 2$)





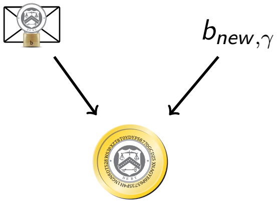
Exchange: Blind sign change (RSA)

1. Take $f'_{new,\gamma}$.
2. Compute $s' := f'^{d}_{new,\gamma} \mod n$.
3. Send signature s'.



Customer: Unblind change (RSA)

1. Receive s'.
2. Compute $s := s' b_{new,\gamma}^{-1} \mod n$.

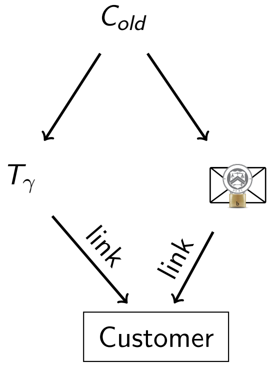


Exchange: Allow linking change

Given C_{old}

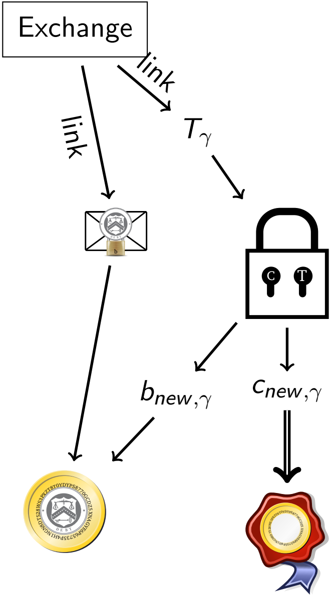
return T_γ and

$$s := s' b_{new,\gamma}^{-1} \mod n.$$



Customer: Link (threat!)

1. Have c_{old}.
2. Obtain T_γ, s from exchange
3. Compute $X_\gamma = c_{old} T_\gamma$
4. Derive $c_{new,\gamma}$ and $b_{new,\gamma}$ from X_γ
5. Unblind $s := s' b_{new,\gamma}^{-1} \mod n$



Refresh protocol summary

- Customer asks exchange to convert old coin to new coin
- Protocol ensures new coins can be recovered from old coin

$\Rightarrow$ New coins are owned by the same entity!

Thus, the refresh protocol allows:

- To give unlinkable change.
- To give refunds to an anonymous customer.
- To expire old keys and migrate coins to new ones.
- To handle protocol aborts.

Transactions via refresh are equivalent to sharing a wallet.

Break

Secure Multiparty Computation (SMC)

- Alice und Bob haben private Daten a_i and b_i.
- Alice und Bob führen ein Protokoll aus und berechnen gemeinsam $f(a_i, b_i)$.
- Nur einer von beiden lernt das Ergebnis (i.d.R.)

Adversary model

Honest but curious

Homomorphic Encryption

$$E(x_1 \oplus x_2) = E(x_1) \otimes E(x_2) \tag{3}$$

Multiplicative Homomorphism: RSA & ElGamal

- Unpadded RSA (multiplicative):

$$E(x_1) \cdot E(x_2) = x_1^e x_2^2 = E(x_1 \cdot x_2) \tag{4}$$

- ElGamal:

$$\begin{aligned} E(x_1) \cdot E(x_2) &= (g^{r_1}, x_1 \cdot h^{r_1})(g^{r_2}, x_2 \cdot h^{r_2}) && (5) \\ &= (g^{r_1+r_2}), (x_1 \cdot x_2) h^{r_1+r_2}) && (6) \\ &= E(m_1 \cdot m_2) && (7) \end{aligned}$$

Additive Homomorphism: Paillier

$$\mathsf{E}_K(m) := g^m \cdot r^n \mod n^2, \tag{8}$$

$$\mathsf{D}_K(c) := \frac{(c^\lambda \mod n^2) - 1}{n} \cdot \mu \mod n \tag{9}$$

where the public key $K = (n, g)$, m is the plaintext, c the ciphertext, n the product of $p, q \in \mathbb{P}$ of equal length, and $g \in \mathbb{Z}^*_{n^2}$. In Paillier, the private key is (λ, μ), which is computed from p and q as follows:

$$\lambda := \mathrm{lcm}(p-1, q-1), \tag{10}$$

$$\mu := \left(\frac{(g^\lambda \mod n^2) - 1}{n} \right)^{-1} \mod n. \tag{11}$$

Paillier offers additive homomorphic public-key encryption, that is:

$$\mathsf{E}_K(a) \otimes \mathsf{E}_K(b) \equiv \mathsf{E}_K(a+b) \tag{12}$$

for any public key K.

Fully homomorphic encryption

Additive:

$$E(A) \oplus E(B) = E(A + B) \tag{13}$$

and multiplicative:

$$E(A) \otimes E(B) = E(A \cdot B) \tag{14}$$

Known cryptosystems: Brakerski-Gentry-Vaikuntanathan (BGV), NTRU, Gentry-Sahai-Waters (GSW).

Break

Example: Secure Scalar Product

- Original idea by Ioannids et al. in 2002 (use: $(a-b)^2 = a^2 - 2ab + b^2$)
- Refined by Amirbekyan et al. in 2007 (corrected math)
- Implemented with practical extensions in GNUnet (negative numbers, small numbers, concrete protocol, set intersection, implementation).

Preliminaries

- Alice has public key A and input map $m_A : M_A \to \mathbb{Z}$.
- Bob has public key B and input map $m_B : M_B \to \mathbb{Z}$.
- We want to calculate

$$\sum_{i \in M_A \cap M_B} m_A(i) m_B(i) \tag{15}$$

- We first calculate $M = M_A \cap M_B$.
- Define $a_i := m_A(i)$ and $b_i := m_B(i)$ for $i \in M$.
- Let s denote a shared static offset.

Network Protocol

- Alice transmits $\mathrm{E}_A(s + a_i)$ for $i \in M$ to Bob.
- Bob creates two random permutations π and π' over the elements in M, and a random vector r_i for $i \in M$ and sends

$$\begin{aligned} R &:= \mathrm{E}_A(s + a_{\pi(i)}) \otimes \mathrm{E}_A(s - r_{\pi(i)} - b_{\pi(i)}) && (16) \\ &= \mathrm{E}_A(2 \cdot s + a_{\pi(i)} - r_{\pi(i)} - b_{\pi(i)}), && (17) \\ R' &:= \mathrm{E}_A(s + a_{\pi'(i)}) \otimes \mathrm{E}_A(s - r_{\pi'(i)}) && (18) \\ &= \mathrm{E}_A(2 \cdot s + a_{\pi'(i)} - r_{\pi'(i)}), && (19) \\ S &:= \sum (r_i + b_i)^2, && (20) \\ S' &:= \sum r_i^2 && (21) \end{aligned}$$

Decryption (1/3)

Alice decrypts R and R' and computes for $i \in M$:

$$a_{\pi(i)} - b_{\pi(i)} - r_{\pi(i)} = \mathsf{D}_A\left(R\right) - 2 \cdot s, \tag{22}$$

$$a_{\pi'(i)} - r_{\pi'(i)} = \mathsf{D}_A\left(R'\right) - 2 \cdot s, \tag{23}$$

which is used to calculate

$$T := \sum_{i \in M} a_i^2 \tag{24}$$

$$U := -\sum_{i \in M} (a_{\pi(i)} - b_{\pi(i)} - r_{\pi(i)})^2 \tag{25}$$

$$U' := -\sum_{i \in M} (a_{\pi'(i)} - r_{\pi'(i)})^2 \tag{26}$$

Decryption (2/3)

She then computes

$$\begin{aligned}
P :&= S + T + U \\
&= \sum_{i \in M} (b_i + r_i)^2 + \sum_{i \in M} a_i^2 + \left(-\sum_{i \in M} (a_i - b_i - r_i)^2 \right) \\
&= \sum_{i \in M} \left((b_i + r_i)^2 + a_i^2 - (a_i - b_i - r_i)^2 \right) \\
&= 2 \cdot \sum_{i \in M} a_i (b_i + r_i). \\
P' :&= S' + T + U' \\
&= \sum_{i \in M} r_i^2 + \sum_{i \in M} a_i^2 + \left(-\sum_{i \in M} (a_i - r_i)^2 \right) \\
&= \sum_{i \in M} \left(r_i^2 + a_i^2 - (a_i - r_i)^2 \right) = 2 \cdot \sum_{i \in M} a_i r_i.
\end{aligned}$$

Decryption (3/3)

Finally, Alice computes the scalar product using:

$$\frac{P - P'}{2} = \sum_{i \in M} a_i(b_i + r_i) - \sum_{i \in M} a_i r_i = \sum_{i \in M} a_i b_i. \qquad (27)$$

Computing Discrete Logarithms

Who said calculating DLOG was hard?

ECC Version[1]

Alice's public key ist $A = g^a$, ihr private key ist a. Alices schickt an Bob $(g_i, h_i) = (g^{r_i}, g^{r_i a + a_i})$ mit zufälligen Werten r_i für $i \in M$. Bob antwortet mit

$$\left(\prod_{i \in M} g_i^{b_i}, \prod_{i \in M} h_i^{b_i}\right) = \left(\prod_{i \in M} g_i^{b_i}, (\prod_{i \in M} g_i^{b_i})^a g^{\sum_{i \in M} a_i b_i}\right)$$

Alice kann dann berechnen

$$\left(\prod_{i \in M} g_i^{b_i}\right)^{-a} \cdot \left(\prod_{i \in M} g_i^{b_i}\right)^{a} \cdot g^{\sum_{i \in M} a_i b_i} = g^{\sum_{i \in M} a_i b_i}.$$

Falls $\sum_{i \in M} a_i b_i$ ausreichend klein ist, kann Alice dann das Skalarprodukt durch Lösung des DLP bestimmen.

[1] Joint work with Tanja Lange

Performance Evaluation

Length	RSA-2048	ECC-2^{20}	ECC-2^{28}
25	14 s	2 s	29 s
50	21 s	2 s	29 s
100	39 s	2 s	29 s
200	77 s	3 s	30 s
400	149 s	OOR	31 s
800	304 s	OOR	33 s
800	3846 kb	OOR	70 kb

The pre-calculation of ECC-2^{28} is $\times 16$ more expensive than for ECC-2^{20} as the table is set to have size $\sqrt{n}$.

Exercise

Implement function to calculate DLOG.