Non-boring cryptography

Christian Grothoff

Berner Fachhochschule

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Learning Objectives

- Blind signatures and applications
- Homomorphic encryption
- Secure multiparty computation (SMC) theory
- Common SMC adversary models
- Specialized protocols for SMC problems
Reminder: RSA

Pick \( p, q \) prime and \( e \) such that

\[
GCD((p - 1)(q - 1), e) = 1
\]  

(1)

- Define \( n = pq \),
- compute \( d \) such that \( ed \equiv 1 \mod (p - 1)(q - 1) \).
- Let \( s := m^d \mod n \).
- Then \( m \equiv s^e \mod n \).
RSA Summary

- Public key: $n, e$
- Private key: $d \equiv e^{-1} \mod \phi(n)$ where 
  $\phi(n) = (p - 1) \cdot (q - 1)$
- Encryption: $c \equiv m^e \mod n$
- Decryption: $m \equiv c^d \mod n$
- Signing: $s \equiv m^d \mod n$
- Verifying: $m \equiv s^e \mod n$?
Low Encryption Exponent Attack

- $e$ is known
- $M$ maybe small
- $C = M^e < n$?
- If so, can compute $M = \sqrt[e]{C}$

$\Rightarrow$ Small $e$ can be bad!
Padding and RSA Symmetry

- Padding can be used to avoid low exponent issues (and issues with \( m = 0 \) or \( m = 1 \))
- Randomized padding defeats chosen plaintext attacks
- Padding breaks RSA symmetry:
  \[
  D_{A_{priv}}(D_{B_{priv}}(E_{A_{pub}}(E_{B_{pub}}(M)))) \neq M
  \]  
  (2)
- PKCS#1 / RFC 3447 define a padding standard
Blind signatures with RSA

1. Obtain public key \((e, n)\)
2. Compute
   \[ f := FDH(m), \]
   \[ f < n. \]
3. Pick blinding factor
   \[ b \in \mathbb{Z}_n \]
4. Transmit
   \[ f' := fb^e \mod n \]
Blind signatures with RSA

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1. Receive $f'$.
2. Compute $s' := f'^d \mod n$.
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Blind signatures with RSA

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1. Receive \(s'\).
2. Compute 
   \(s := s'b^{-1} \mod n\)
What domain of digital communication should we be most concerned about?
Surveillance concerns

- Everybody knows about Internet surveillance.
- But is it *that* bad?
Surveillance concerns

- Everybody knows about Internet surveillance.
- But is it **that** bad?
  - You can choose when and where to use the Internet
  - You can anonymously access the Web using Tor
  - You can find open access points that do not require authentication
  - IP packets do not include your precise location or name
  - ISPs typically store this meta data for days, weeks or months
Where is it worse?

This was a question posed to RAND researchers in 1971:

“Suppose you were an advisor to the head of the KGB, the Soviet Secret Police. Suppose you are given the assignment of designing a system for the surveillance of all citizens and visitors within the boundaries of the USSR. The system is not to be too obtrusive or obvious. What would be your decision?”

“I think one of the big things that we need to do, is we need to get a way from true-name payments on the Internet. The credit card payment system is one of the worst things that happened for the user, in terms of being able to divorce their access from their identity.” –Edward Snowden, IETF 93 (2015)
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What is worse:

- When you pay by CC, the information includes your name
- When you pay in person with CC, your location is also known
- You often have no alternative payment methods available
- You hardly ever can use someone else’s CC
- Anonymous prepaid cards are difficult to get and expensive
- Payment information is typically stored for at least 6 years
Banks have Problems, too!

3D secure (“verified by visa”) is a nightmare:

- Complicated process
- Shifts liability to consumer
- Significant latency
- Can refuse valid requests
- Legal vendors excluded
- No privacy for buyers

Online credit card payments will be replaced, but with what?
The Bank’s Problem

- Global tech companies push oligopolies
- Privacy and federated finance are at risk
- Economic sovereignty is in danger
Predicting the Future

▶ Google and Apple will be your bank and run your payment system
▶ They can target advertising based on your purchase history, location and your ability to pay
▶ They will provide more usable, faster and broadly available payment solutions; our federated banking system will be history
▶ After they dominate the payment sector, they will start to charge fees befitting their oligopoly size
▶ Competitors and vendors not aligning with their corporate “values” will be excluded by policy and go bankrupt
▶ The imperium will have another major tool for its financial warfare
Do you want to live under total surveillance?
Banking, Surveillance and Physical Security

https://www.youtube.com/watch?v=GyJZVif2Vx (4'2019)
Break
Bitcoin

Payment system using a block chain:

▶ Public keys identify accounts, private keys used to send money from the account into other accounts.
▶ Set of internally consistent transactions form block
▶ Each block includes a transaction creating fresh coins and transferring applicable fees to block creator
▶ Difficulty adjusts to mining power to mine a block in $\approx 10$ minutes
▶ Amount of coins created per block is exponentially decreasing
Mining

Mining requires:

- Learning pending transactions from peers
- Selecting a subset of transactions which is valid (no double spending) by computing current account balances against the entire history
- Finding a hash collision (with adaptive difficulty)
- Propagating the new block to other miners

Usually specialized systems are used for finding hash collisions.
The CAP Theorem

We like three properties in distributed systems:

- Consistency: everybody agrees on the state
- Availability: the system is always available
- Partition tolerance: the system handles component (host, link) failure, including network partitions

CAP Theorem is an **impossibility proof**: you can only have 2/3.
CAP & Bitcoin

Bitcoin is inconsistent:

- Conflicting blocks can be mined at the same time
- This can happen by accident, or on purpose!
- Coins could be spent twice, once on each fork of the chain!
- Longest chain is considered “valid”
- Original paper suggests to consider transaction confirmed only after at least 6 blocks past the transaction.

⇒ Bitcoin is not consistent.
⇒ Competitively long alternative chains void durability even after 6 blocks!
Bitcoin performance

- Privacy: all transactions happen in the clear in public view
- Latency: transactions take 1h to kind-of be confirmed
- Storage: grows linearly forever, no garbage collection
- Power: mining consumes more than the entire state of Denmark today
- Rate: Network handles at most about 7 transactions per second
- Accountability: use of public keys as addresses enables criminal use

⇒ Bitcoin fever lasting for years. Why?
Altcoins

- Dogecoin: same as Bitcoin, just named after a dog meme (an idea that is obviously worth billions!)
- Zcash: uses ZKSNARKs\textsuperscript{1} to hide transactions (criminal activity on Bitcoin was too low)
- Ethereum: run Turing-complete virtual machine logic in the blockchain to enable “smart” contracts and arbitrary applications, not just payments (is “Accelerando” an utopia or dystopia?)

Experimental designs promising to drastically improve performance (Bolt, Lightning) have so far failed to deliver.

\textsuperscript{1}≈ 1-15 minutes CPU time to create new transaction needed!
Case study: Private payments

“A company is developing new software for private payments. This will enable its customers to transact with “complete” privacy (like cash). The solution does not include backdoors, and thus the company cannot block payments to support trade embargos or anti money laundering efforts.”

Discuss virtues and vices affected.

Does it make a difference if the software is Free Software developed by a community instead of proprietary software from a company?

Suppose the company added a feature to provide income transparency where the state gets to see who receives funds (but not who made payments). Does this change your assessment?
Break
GNU Taler

Digital cash, made socially responsible.

Privacy-Preserving, Practical, Taxable, Free Software, Efficient
What is Taler?

Taler is an electronic instant payment system.

- Uses electronic coins stored in **wallets** on customer’s device
- Like **cash**
- Pay in **existing currencies** (i.e. EUR, USD, BTC), or use it to create new **regional currencies**
Design goals for the GNU Taler Payment System

GNU Taler must ...

1. ... be implemented as free software.
2. ... protect the privacy of buyers.
3. ... must enable the state to tax income and crack down on illegal business activities.
4. ... prevent payment fraud.
5. ... only disclose the minimal amount of information necessary.
6. ... be usable.
7. ... be efficient.
8. ... avoid single points of failure.
9. ... foster competition.
Taler Overview

- Customer
  - Withdraw coins
  - Spend coins

- Exchange
  - Verify

- Auditor
  - Deposit coins

- Merchant
Architecture of Taler

1. Pay exchange
2. Wire transfer
3. Withdraw coins
4. Spend coins
5. Deposit coins
6. Wire transfer
7. View balance
Architecture of Taler

⇒ Convenient, taxable, privacy-enhancing, & resource friendly!
Usability of Taler

https://demo.taler.net/

1. Install Web extension.
2. Visit the bank.demo.taler.net to withdraw coins.
3. Visit the shop.demo.taler.net to spend coins.
Taxability

We say Taler is taxable because:

- Merchant’s income is visible from deposits.
- Hash of contract is part of deposit data.
- State can trace income and enforce taxation.
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Limitations:

- withdraw loophole
- *sharing* coins among family and friends
How does it work?

We use a few ancient constructions:

- Cryptographic hash function (1989)
- Blind signature (1983)
- Schnorr signature (1989)
- Diffie-Hellman key exchange (1976)
- Cut-and-choose zero-knowledge proof (1985)

But of course we use modern instantiations.
Exchange setup: Create a denomination key (RSA)

1. Pick random primes $p, q$.
2. Compute $n := pq$, 
   \[\phi(n) = (p - 1)(q - 1)\]
3. Pick small $e < \phi(n)$ such that $d := e^{-1} \mod \phi(n)$ exists.
4. Publish public key $(e, n)$. 
Merchant: Create a signing key (EdDSA)

- pick random $m \mod o$ as private key
- $M = mG$ public key

Capability: $m \Rightarrow M$
Customer: Create a planchet (EdDSA)

- Pick random $c \mod o$ private key
- $C = cG$ public key

Capability: $c \Rightarrow$
1. Obtain public key \((e, n)\)
2. Compute \(f := FDH(C)\), \(f < n\).
3. Pick blinding factor \(b \in \mathbb{Z}_n\)
4. Transmit \(f' := fb^e \mod n\)
1. Receive $f'$.  
2. Compute $s' := f'^d \mod n$.  
3. Send signature $s'$. 

Customer
Customer: Unblind coin (RSA)

1. Receive $s'$.
2. Compute $s := s'b^{-1} \mod n$
Withdrawing coins on the Web

1. User authentication
2. Send account portal
3. Initiate withdrawal (specify amount and exchange)
4. Request coin denomination keys and wire transfer data
5. Send coin denomination keys and wire transfer data
6. Execute withdrawal
7. Request transaction authorization (optional)
8. Transaction authorization
9. Withdrawal confirmation
10. Execute wire transfer
11. Withdraw request
12. Signed blinded coins
13. Unblind coins

Customer Browser

Bank Site

Taler (Withdraw coins)

Taler Exchange

HTTP

HTTPS

Wire transfer
Customer: Build shopping cart

Merchant
Merchant: Propose contract (EdDSA)

1. Complete proposal $D$.
2. Send $D$, $EdDSA_m(D)$
Customer: Spend coin (EdDSA)

1. Receive proposal $D$, $\text{EdDSA}_m(D)$.
2. Send $s$, $C$, $\text{EdDSA}_c(D)$
Merchant and Exchange: Verify coin (RSA)

\[ s^e \equiv m \mod n \]

\[ \iff \]
Payment processing with Taler

1. Choose goods by navigating to offer URL
2. Send signed digital contract proposal
3. Select Taler payment method (skippable with auto-detection)
4. Affirm contract
5. Navigate to fulfillment URL
6. Send hash of digital contract and payment information
7. Send payment
8. Forward payment
9. Confirm payment
10. Confirm payment
11. Reload fulfillment URL for delivery
12. Provide product resource
Warranting deposit safety

Exchange has another online signing key $W = wG$:

Sends $E, \text{EdDSA}_w(M, H(D), \text{FDH}(C))$ to the merchant.

This signature means that $M$ was the first to deposit $C$ and that the exchange thus must pay $M$.

Without this, an evil exchange could renege on the deposit confirmation and claim double-spending if a coin were deposited twice, and then not pay either merchant!
Online keys

- The exchange needs $d$ and $w$ to be available for online signing.
- The corresponding public keys $W$ and $(e, n)$ are certified using Taler’s public key infrastructure (which uses offline-only keys).

What happens if those private keys are compromised?
Denomination key \((e, n)\) compromise

- An attacker who learns \(d\) can sign an arbitrary number of illicit coins into existence and deposit them.
- Auditor and exchange can detect this once the total number of deposits (illicit and legitimate) exceeds the number of legitimate coins the exchange created.
- At this point, \((e, n)\) is revoked. Users of unspent legitimate coins reveal \(b\) from their withdrawal operation and obtain a refund.
- The financial loss of the exchange is bounded by the number of legitimate coins signed with \(d\).
- Taler frequently rotates denomination signing keys and deletes \(d\) after the signing period of the respective key expires.
Online signing key $W$ compromise

- An attacker who learns $w$ can sign deposit confirmations.
- Attacker sets up two (or more) merchants and customer(s) which double-spend legitimate coins at both merchants.
- The merchants only deposit each coin once at the exchange and get paid once.
- The attacker then uses $w$ to fake deposit confirmations for the double-spent transactions.
- The attacker uses the faked deposit confirmations to complain to the auditor that the exchange did not honor the (faked) deposit confirmations.

The auditor can then detect the double-spending, but cannot tell who is to blame, and (likely) would presume an evil exchange, forcing it to pay both merchants.
Break
Giving change

It would be inefficient to pay EUR 100 with 1 cent coins!

- Denomination key represents value of a coin.
- Exchange may offer various denominations for coins.
- Wallet may not have exact change!
- Usability requires ability to pay given sufficient total funds.
Giving change

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Key goals:

- maintain unlinkability
- maintain taxability of transactions
Giving change

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Method:

- Contract can specify to only pay *partial value* of a coin.
- Exchange allows wallet to obtain *unlinkable change* for remaining coin value.
Diffie-Hellman (ECDH)

1. Create private keys $c$, $t$ mod $o$
2. Define $C = cG$
3. Define $T = tG$
4. Compute DH  
   $cT = c(tG) = t(cG) = tC$
Strawman solution

Given partially spent private coin key $c_{old}$:

1. Pick random $c_{new}$ mod $o$ private key
2. $C_{new} = c_{new} G$ public key
3. Pick random $b_{new}$
4. Compute $f_{new} := FDH(C_{new}), m < n.$
5. Transmit $f'_{new} := f_{new} b_{new}^e \mod n$

... and sign request for change with $c_{old}$. 
Strawman solution

Given partially spent private coin key $c_{old}$:

1. Pick random $c_{new}$ mod $o$ private key
2. $C_{new} = c_{new}G$ public key
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4. Compute $f_{new} := FDH(C_{new})$, $m < n$.
5. Transmit $f'_{new} := f_{new}b_{new}^e$ mod $n$

... and sign request for change with $c_{old}$.

Problem: Owner of $c_{new}$ may differ from owner of $c_{old}$!
Given partially spent private coin key $c_{old}$:

1. Let $C_{old} := c_{old} G$ (as before)
2. Create random private transfer key $t \mod o$
3. Compute $T := tG$
4. Compute 
   \[ X := c_{old}(tG) = t(c_{old} G) = tC_{old} \]
5. Derive $c_{new}$ and $b_{new}$ from $X$
6. Compute $C_{new} := c_{new} G$
7. Compute $f_{new} := FDH(C_{new})$
8. Transmit $f'_{new} := f_{new} b_{new}$
Cut-and-Choose

c_{old} \xrightarrow{t_1} c_{new,1} \xrightarrow{b_{new,1}} c_{new,2} \xrightarrow{b_{new,2}} c_{new,3} \xrightarrow{b_{new,3}}

transmit

Exchange
Exchange: Choose!

Exchange sends back random $\gamma \in \{1, 2, 3\}$ to the customer.
1. If $\gamma = 1$, send $t_2, t_3$ to exchange
2. If $\gamma = 2$, send $t_1, t_3$ to exchange
3. If $\gamma = 3$, send $t_1, t_2$ to exchange
Exchange: Verify ($\gamma = 2$)
1. Take $f'_{\text{new},\gamma}$.
2. Compute $s' := f'^d_{\text{new},\gamma}$ mod $n$.
3. Send signature $s'$.
Customer: Unblind change (RSA)

1. Receive $s'$.
2. Compute $s := s' b_{new, \gamma}^{-1}$ mod $n$. 

\[ \text{XNAGYE6P65735P4H1NGN8DT528W} \]
\[ \text{S3PXZT8T0YDYPS8770GCDZ5} \]
Exchange: Allow linking change

Given $C_{old}$

return $T_\gamma$ and

$s := s' b_{new,\gamma}^{-1} \mod n$. 

```
C_{old}   
  ↓      
  ↓      
T_\gamma
  ↓      
  ↓      
link  link
  ↓      
Customer
```
Customer: Link (threat!)

1. Have $c_{old}$.
2. Obtain $T_\gamma$, $s$ from exchange
3. Compute $X_\gamma = c_{old} \cdot T_\gamma$
4. Derive $c_{new,\gamma}$ and $b_{new,\gamma}$ from $X_\gamma$
5. Unblind $s := s' \cdot b_{new,\gamma}^{-1}$ mod $n$
Refresh protocol summary

- Customer asks exchange to convert old coin to new coin
- Protocol ensures new coins can be recovered from old coin
  ⇒ New coins are owned by the same entity!

Thus, the refresh protocol allows:
- To give unlinkable change.
- To give refunds to an anonymous customer.
- To expire old keys and migrate coins to new ones.
- To handle protocol aborts.

Transactions via refresh are equivalent to sharing a wallet.
Example: The Taler Snack Machine

Integration of a MDB/ICP to Taler gateway.
Implementation of a NFC or QR-Code to Taler wallet interface.

By M. Boss and D. Hofer
Software architecture for the Taler Snack Machine

Application

Raspbian

Raspberry Pi

TCP/IP

MDB  NFC  TFT
User story: Install App on Android

https://wallet.taler.net/
User story: Withdraw e-cash
User story: Use machine!
Summary

▶ We can design protocols that fail soft.
▶ GNU Taler’s design limits financial damage even in the case private keys are compromised.
▶ GNU Taler does:
  ▶ Gives change, can provide refunds
  ▶ Integrates nicely with HTTP, handles network failures
  ▶ High performance
  ▶ Free Software
  ▶ Formal security proofs
Next Steps

- Implementation still needs:
  - Escrowed wallet backup and synchronization solution ⇒ Anastasis!
  - Finish integration with existing banking system (EBICS, FinTS) ⇒ LibFinTS
  - Code security audit ⇒ Exchange report now public!
  - Improved design and usability ⇒ Discussions with UX experts on public list!
  - Internationalization ⇒ https://weblate.taler.net/
  - Porting to more platforms (Web shops, Android, POS, iOS)

- Regulatory approval (withdraw and deposit limits, independent auditor, KYC/AML process validation)

- In discussions with various (commercial and central) banks
Visions

- Be paid to read advertising, starting with spam
- Give welfare without intermediaries taking huge cuts
- Forster regional trade via regional currencies
- Eliminate corruption by making all income visible
- Stop the mining by making crypto-currencies useless for anything but crime
Break
Secure Multiparty Computation (SMC)

- Alice und Bob haben private Daten $a_i$ and $b_i$.
- Alice und Bob führen ein Protokoll aus und berechnen gemeinsam $f(a_i, b_i)$.
- Nur einer von beiden lernt das Ergebnis (i.d.R.)
Adversary model

Honest but curious
Homomorphic Encryption

\[ E(x_1 \oplus x_2) = E(x_1) \otimes E(x_2) \]  (3)
Multiplicative Homomorphism: RSA & ElGamal

Unpadded RSA (multiplicative):

\[
E(x_1) \cdot E(x_2) = x_1^e x_2^e = E(x_1 \cdot x_2)
\]  
(4)

ElGamal:

\[
E(x_1) \cdot E(x_2) = (g^{r_1}, x_1 \cdot h^{r_1})(g^{r_2}, x_2 \cdot h^{r_2})
\]  
(5)
\[
= (g^{r_1+r_2}, (x_1 \cdot x_2)h^{r_1+r_2})
\]  
(6)
\[
= E(m_1 \cdot m_2)
\]  
(7)
Additive Homomorphism: Paillier

\[ E_K(m) := g^m \cdot r^n \mod n^2, \quad (8) \]

\[ D_K(c) := \left( \frac{c^\lambda \mod n^2 - 1}{n} \cdot \mu \mod n \right) \quad (9) \]

where the public key \( K = (n, g) \), \( m \) is the plaintext, \( c \) the ciphertext, \( n \) the product of \( p, q \in \mathbb{P} \) of equal length, and \( g \in \mathbb{Z}_{n^2}^* \). In Paillier, the private key is \((\lambda, \mu)\), which is computed from \( p \) and \( q \) as follows:

\[ \lambda := \text{lcm}(p - 1, q - 1), \quad (10) \]

\[ \mu := \left( \frac{(g^\lambda \mod n^2 - 1)}{n} \right)^{-1} \mod n \]. \quad (11) \]

Paillier offers additive homomorphic public-key encryption, that is:

\[ E_K(a) \otimes E_K(b) \equiv E_K(a + b) \quad (12) \]

for any public key \( K \).
Fully homomorphic encryption

Additive:
\[ E(A) \oplus E(B) = E(A + B) \]  \hspace{1cm} (13)

and multiplicative:
\[ E(A) \otimes E(B) = E(A \cdot B) \]  \hspace{1cm} (14)

Known cryptosystems: Brakerski-Gentry-Vaikuntanathan (BGV), NTRU, Gentry-Sahai-Waters (GSW).
Break
Example: Secure Scalar Product

- Original idea by Ioannids et al. in 2002 (use: \((a - b)^2 = a^2 - 2ab + b^2\))
- Refined by Amirbekyan et al. in 2007 (corrected math)
- Now providing protocol with practical extensions (negative numbers, small numbers, set intersection).
Preliminaries

- Alice has public key $A$ and input map $m_A : M_A \rightarrow \mathbb{Z}$.
- Bob has public key $B$ and input map $m_B : M_B \rightarrow \mathbb{Z}$.
- We want to calculate
  \[ \sum_{i \in M_A \cap M_B} m_A(i)m_B(i) \]  
  (15)
- We first calculate $M = M_A \cap M_B$.
- Define $a_i := m_A(i)$ and $b_i := m_B(i)$ for $i \in M$.
- Let $s$ denote a shared static offset.
Alice transmits $E_A(s + a_i)$ for $i \in M$ to Bob.

Bob creates two random permutations $\pi$ and $\pi'$ over the elements in $M$, and a random vector $r_i$ for $i \in M$ and sends

$$R : = E_A(s + a_{\pi(i)}) \otimes E_A(s - r_{\pi(i)} - b_{\pi(i)})$$

$$= E_A(2 \cdot s + a_{\pi(i)} - r_{\pi(i)} - b_{\pi(i)}),$$

$$R' : = E_A(s + a_{\pi'(i)}) \otimes E_A(s - r_{\pi'(i)})$$

$$= E_A(2 \cdot s + a_{\pi'(i)} - r_{\pi'(i)}),$$

$$S : = \sum (r_i + b_i)^2,$$

$$S' : = \sum r_i^2$$
Decryption (1/3)

Alice decrypts $R$ and $R'$ and computes for $i \in M$:

$$a_{\pi}(i) - b_{\pi}(i) - r_{\pi}(i) = D_A(R) - 2 \cdot s,$$

$$a_{\pi'}(i) - r_{\pi'}(i) = D_A(R') - 2 \cdot s,$$

which is used to calculate

$$T := \sum_{i \in M} a_i^2$$

$$U := -\sum_{i \in M} (a_{\pi}(i) - b_{\pi}(i) - r_{\pi}(i))^2$$

$$U' := -\sum_{i \in M} (a_{\pi'}(i) - r_{\pi'}(i))^2$$
Decryption (2/3)

She then computes

\[ P := S + T + U \]

\[ = \sum_{i \in M} (b_i + r_i)^2 + \sum_{i \in M} a_i^2 + \left( -\sum_{i \in M} (a_i - b_i - r_i)^2 \right) \]

\[ = \sum_{i \in M} ((b_i + r_i)^2 + a_i^2 - (a_i - b_i - r_i)^2) \]

\[ = 2 \cdot \sum_{i \in M} a_i (b_i + r_i). \]

\[ P' := S' + T + U' \]

\[ = \sum_{i \in M} r_i^2 + \sum_{i \in M} a_i^2 + \left( -\sum_{i \in M} (a_i - r_i)^2 \right) \]

\[ = \sum_{i \in M} (r_i^2 + a_i^2 - (a_i - r_i)^2) = 2 \cdot \sum_{i \in M} a_i r_i. \]
Finally, Alice computes the scalar product using:

\[ \frac{P - P'}{2} = \sum_{i \in M} a_i (b_i + r_i) - \sum_{i \in M} a_i r_i = \sum_{i \in M} a_i b_i. \] (27)
Computing Discrete Logarithms

Who said calculating DLOG was hard?
Alice's public key ist $A = g^a$, ihr private key ist $a$. Alices schickt an Bob $(g_i, h_i) = (g^{r_i}, g^{r_i+a_i})$ mit zufälligen Werten $r_i$ für $i \in M$. Bob antwortet mit

$$
\left( \prod_{i \in M} g_i^{b_i}, \prod_{i \in M} h_i^{b_i} \right) = \left( \prod_{i \in M} g_i^{b_i}, (\prod_{i \in M} g_i^{b_i})^a g^{\sum_{i \in M} a_i b_i} \right)
$$

Alice kann dann berechnen

$$
\left( \prod_{i \in M} g_i^{b_i} \right)^{-a} \cdot \left( \prod_{i \in M} g_i^{b_i} \right)^a \cdot g^{\sum_{i \in M} a_i b_i} = g^{\sum_{i \in M} a_i b_i}.
$$

Falls $\sum_{i \in M} a_i b_i$ ausreichend klein ist, kann Alice dann das Skalarprodukt durch Lösung des DLP bestimmen.

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4 Joint work with Tanja Lange
The performance evaluation of different encryption methods is shown in the table below:

<table>
<thead>
<tr>
<th>Length</th>
<th>RSA-2048</th>
<th>ECC-2^{20}</th>
<th>ECC-2^{28}</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>14 s</td>
<td>2 s</td>
<td>29 s</td>
</tr>
<tr>
<td>50</td>
<td>21 s</td>
<td>2 s</td>
<td>29 s</td>
</tr>
<tr>
<td>100</td>
<td>39 s</td>
<td>2 s</td>
<td>29 s</td>
</tr>
<tr>
<td>200</td>
<td>77 s</td>
<td>3 s</td>
<td>30 s</td>
</tr>
<tr>
<td>400</td>
<td>149 s</td>
<td>OOR</td>
<td>31 s</td>
</tr>
<tr>
<td>800</td>
<td>304 s</td>
<td>OOR</td>
<td>33 s</td>
</tr>
<tr>
<td>800</td>
<td>3846 kb</td>
<td>OOR</td>
<td>70 kb</td>
</tr>
</tbody>
</table>

The pre-calculation of ECC-2^{28} is $\times 16$ more expensive than for ECC-2^{20} as the table is set to have size $\sqrt{n}$. 
Exercise

Implement function to calculate DLOG.