NEXT GENERATION INTERNET

Blind Signatures

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31.05.2024

Learning Objectives

How do standard RSA signatures work?

What are Blind Signatures?

How do RSA blind signatures work?

What are the main applications for Blind Signatures?

Reminder: RSA

Generate random p, q primes and e such that

$$GCD((p-1)(q-1), e) = 1 \tag{1}$$

- Define $n = pq$,
- compute d such that $ed \equiv 1 \mod (p-1)(q-1)$.
- Let $s := m^d \mod n$.
- Then $m \equiv s^e \mod n$.

RSA Summary

- Public key: n, e
- Private key: $d \equiv e^{-1} \mod \phi(n)$ where $\phi(n) = (p - 1) \cdot (q - 1)$
- Encryption: $c \equiv m^e \mod n$
- Decryption: $m \equiv c^d \mod n$
- Signing: $s \equiv m^d \mod n$
- Verifying: $m \equiv s^e \mod n$?

These equations are heavily simplified and should not be used like this in production!

Low Encryption Exponent Attack

- e is known
- m maybe small
- $C = m^e < n$?
- If so, can compute $m = \sqrt[e]{C}$

$\Rightarrow$ Small e can be bad!

Padding and RSA Symmetry

- Padding can be used to avoid low exponent issues (and issues with $m = 0$ or $m = 1$)
- Randomized padding defeats chosen plaintext attacks
- Padding breaks RSA symmetry:

$$D_{A_{priv}}(D_{B_{priv}}(E_{A_{pub}}(E_{B_{pub}}(m)))) \neq m \tag{2}$$

- PKCS#1 / RFC 3447 define a padding standard

Blind signatures with RSA [1]

1. Obtain public key (e, n)
2. Compute $f := FDH_n(m)$, $f < n$.
3. Generate random blinding factor $b \in \mathbb{Z}_n$
4. Transmit $f' := fb^e \mod n$

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3. Send s'.

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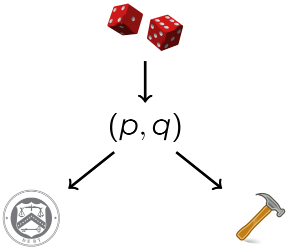
Applications for Blind Signatures

- Untraceable payments
- Unlinkable access tokens (PrivacyPass)

Provider setup: Create a denomination key (RSA)

1. Generates random primes p, q.
2. Computes $n := pq$,
   $\phi(n) = (p-1)(q-1)$
3. Picks small $e < \phi(n)$ such that
   $d := e^{-1} \mod \phi(n)$ exists.
4. Publishes public key (e, n).



Merchant setup: Create a signing key (EdDSA)

- Generates random $m \mod o$ as private key
- Computes public key $M := mG$

Capability: $m \Rightarrow$ 

$\downarrow$

m

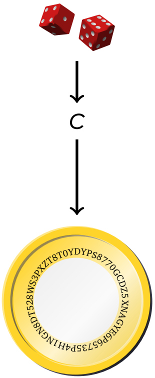
$\downarrow$

M

Customer: Create a planchet (EdDSA)

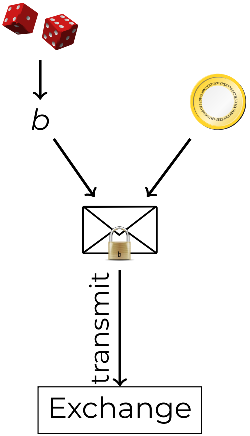
- Generates random $c \mod o$ as private key
- Computes public key $C := cG$

Capability: $c \Rightarrow$ 



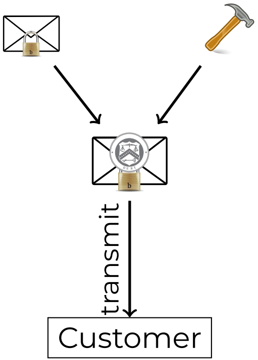
Customer: Blind planchet (RSA)

1. Obtains public key (e, n)
2. Computes $f := FDH_n(C), f < n$.
3. Generates random blinding factor $b \in \mathbb{Z}_n$
4. Transmits $f' := fb^e \mod n$



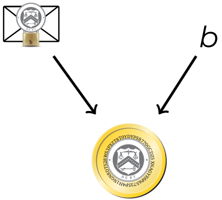
Provider: Blind sign (RSA)

1. Receives f'.
2. Computes $s' := f'^d \mod n$.
3. Sends signature s'.

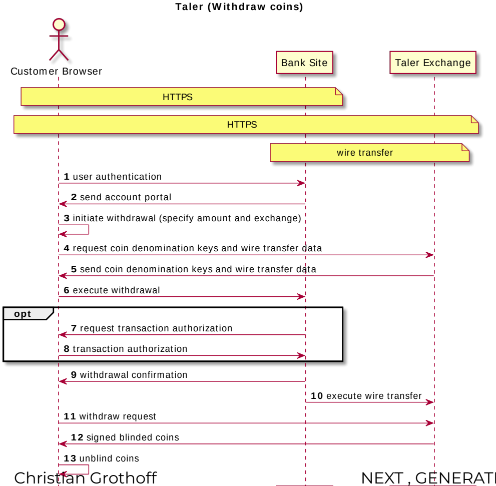


Customer: Unblind signature (RSA)

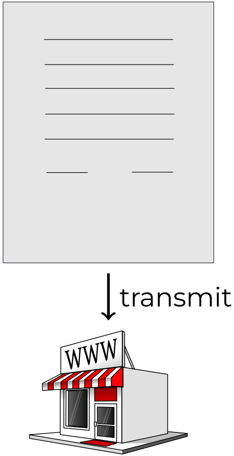
1. Receives s'.
2. Computes $s := s'b^{-1} \mod n$



Withdrawing coins on the Web

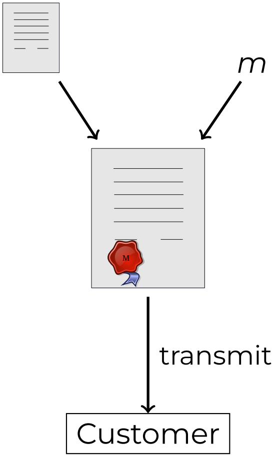


Customer: Build shopping cart

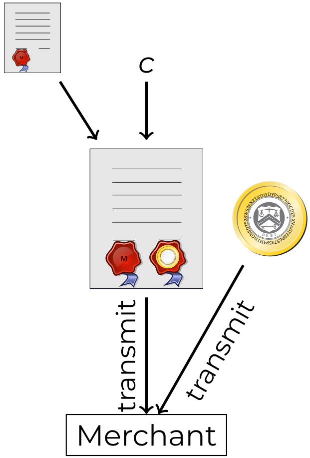


Merchant: Propose contract (EdDSA)

1. Complete proposal D.
2. Send D, $EdDSA_m(D)$



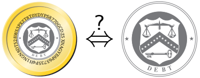
Customer: Spend coin (EdDSA)



1. Receive proposal D, $EdDSA_m(D)$.
2. Send s, C, $EdDSA_c(D)$

Merchant and Provider: Verify coin (RSA)

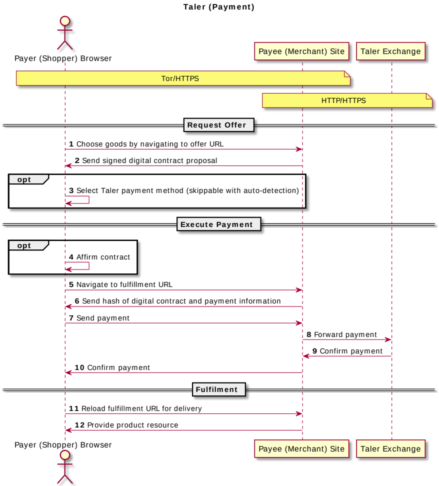
$$s^e \mod n \stackrel{?}{\equiv} FDH_n(C)$$



The provider (Taler: exchange) does not only verify the signature, but also checks that the coin was not double-spent.

GNU Taler is an online payment system.

Payment processing with Blind Signatures



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Acknowledgements

Funded by the European Union (Project 101135475).

Co-funded by
the European Union

Funded by SERI (HEU-Projekt 101135475-TALER).

Project funded by

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Swiss Confederation

Federal Department of Economic Affairs,
Education and Research EAER
State Secretariat for Education,
Research and Innovation SERI

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